Parameters Identification of Controlled Inverted Pendulum Model for a Particular Person

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Abstract

In this work an model inverted pendulum with a single degree of freedom controlled by proportional, integral and differential (PID) controller is used in order to simulate the vibration of center of pressure (COP) of a human body in forward-backward direction during still standing. Numerical experiments using data of typical adult male and different disturbance signals revealed that identified model parameters set coincide with the set reported in the literature which is able to reproduce realistic COP signals and did not depend on the particular sample of a disturbance signal. In this work parameter identification of the model for a particular person is based on an optimal control technique.

Keywords: controlled inverted pendulum model, human posture, still standing, parameter identification, optimal control, disturbance excitation

1. Introduction

One of the most popular models used to simulate the vibration of center of pressure (COP) signal of a human body is a model of inverted pendulum [1-5] controlled by proportional, integral and differential (PID) controller [1,2,4]. In this work the model is referred to as a controlled inverted pendulum (CIP) model. CIP model represents movements of COP in forward-backward direction during still standing. According to Peterka [4] the advantage of CIP model is the ability to explain the movement of COP in physiologically meaningful terms.

In the model the equilibrium of a standing person is maintained by means of forces appearing due to the displacement of the center of mass (COM) of the body (proportional component of PID controller), speed of the COM (differential component of PID controller) and integral of past displacements of COM (integral component of PID controller).

There were no attempts reported so far which intended to identify the CIP model parameters of the particular person and the disturbance torque signal which was present within the human body during the physical experiment from the COP data. In this paper a formal method, which employs the optimum control technique is presented in order to identify the CIP models' parameters and the disturbance torque signal from the COP signal recorded during physical experiment. Obtained data are compared with earlier investigation done by Peterka [4].

2. Methods

2.1. Controlled Inverted Pendulum

In this work the vibration of COP of a human body in forward-backward direction during still standing is generated using an inverted pendulum model with a single degree of freedom (DOF) controlled by PID controller (Fig. 1):

$$I\ddot{u}(t) - mghu(t) = w(u, \dot{u}, \mathbf{p}) \tag{1}$$

where: m - body mass; I – mass moment of inertia of the body around the ankle joint; h – distance of the centre of mass (COM) from the ankle joint; u - sway angle of COM; g – gravitational acceleration; w – non-linear force vector:

$$w(u, \dot{u}, \mathbf{p}) = T_d(t) - T_c(t);$$

where: T_d - cumulative disturbance torque, which generates within the body due to physical condition or sickness; T_c - corrective torque implemented as a PID controller:

$$T_c(t) = K_P u(t) + K_I \int_t u(t)dt + K_D \dot{u}(t);$$

$\mathbf{p} = \{K_P, K_I, K_D\}$ – parameters of the CIP model.

The position of COP, which is measured during experiments, is calculated from sway angle of COM u according to Peterka [4]:

$$u_{COP}(t) = hu(t) - \frac{I\ddot{u}(t)}{mg} = au(t) - b\ddot{u}(t)$$
$$a = h; b = \frac{I}{mg} \tag{2}$$

2.2. Optimum control technique

2.2.1. Error function formulation

A quantitative measure of the deviation of the model behaviour from the available experimental record can be introduced by means of the error function J as

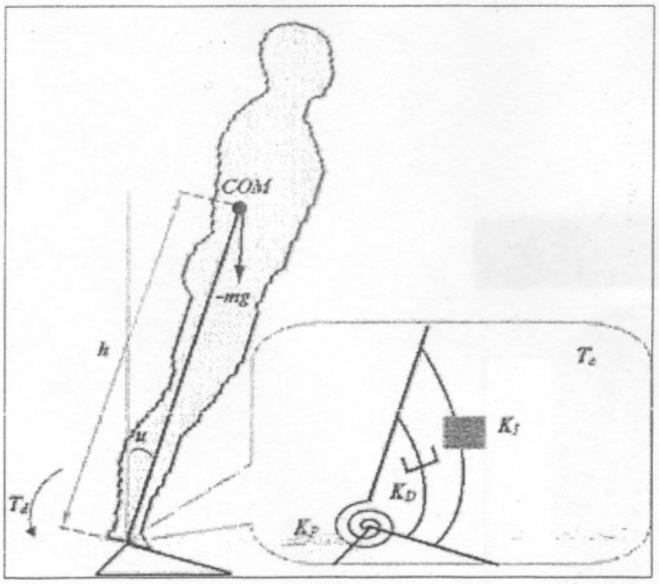

Fig. 1. CIP model of a human posture in forward-backward direction

$$J = \int_0^T \psi(u, \dot{u}, \ddot{u}, \mathbf{p})dt \tag{3}$$

In this work the error function J formulated in terms of the disturbance torque time derivative (Eq. (4)).

where: $$\psi(u, \dot{u}, \ddot{u}, \mathbf{p}) = \frac{1}{2}\int_0^T \left(\dot{T}_d(t)\right)^2 dt; \tag{4}$$

2.2.2. General case of error function minimization

The error function J minimization is performed by using the steepest descent method [12], which is based on the sensitivity functions [10]. The application of the method to CIP model parameters identification was presented by Barauskas and Krušinskienė in [11].

Consider the dynamic system as

$$m\ddot{u} + c\dot{u} + ku = w(u, \dot{u}, \mathbf{p})$$

where vector $\mathbf{p}$ contains the parameters of the model.

The sensitivity function vector of the error function J reads as $\frac{\partial J}{\partial \mathbf{p}}$. The vector represents the search direction in the space of parameters $\mathbf{p}$. In order to obtain $\frac{\partial J}{\partial \mathbf{p}}$, we introduce time derivatives of conjugate variables and express the variation of the error function δJ in terms of $\delta \mathbf{p}$. The basic variation relation reads as:

$$\delta J = \int_0^T (\lambda + \dot{\mu} + \ddot{\eta})\left(\frac{\partial w(u_T, \dot{u}_T, p)}{\partial \mathbf{p}}\delta \mathbf{p}\right)dt \tag{5}$$

Time laws of conjugate variables and their time derivatives are obtained by time integration of the conjugate differential equations

$$\begin{cases} \ddot{\lambda}m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \dot{\eta}\dot{\tilde{k}} = \dfrac{\partial \psi}{\partial u}; \\ \ddot{\mu}m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\dot{\tilde{k}} = -\dfrac{\partial \psi}{\partial \dot{u}}; \\ \ddot{\eta}m - \dot{\eta}\tilde{c} + \eta\tilde{k} = \dfrac{\partial \psi}{\partial \ddot{u}}. \end{cases} \tag{6}$$

With the following boundary conditions:

$$\lambda_T = \dot{\lambda}_T = \dot{\mu}_T = \dot{\eta}_T = \mu_T = \eta_T \tag{7}$$

where:

$$\tilde{c} = c - \frac{\partial w(u, \dot{u}, p)}{\partial \dot{u}};$$
$$\tilde{k} = k - \frac{\partial w(u, \dot{u}, p)}{\partial u}.$$

And in its' turn the disturbance torque signal T_d from the CIP model equation (Eq. 1) reads as:

$$\begin{aligned} &T_d(t) = w(u, \dot{u}, \ddot{u}, \mathbf{p}) \\ &w(u, \dot{u}, \ddot{u}, \mathbf{p}) = I\ddot{u}(t) + K_D\dot{u}(t) + (K_P - mgh)u(t) + K_I\int_0^t u(t)dt \end{aligned} \tag{8}$$

The presented disturbance torque signal generation method (Eq. 8) could be used for T_d identification of a particular person only when the set of CIP model's parameters $\mathbf{p}$ is known. Moreover, T_d can be reconstructed for any set of parameters $\mathbf{p}$.

2.2.2. Minimization of the error function based on time derivative of disturbance torque signal

Let us rewrite the parameters identification algorithm when the error function J is formulated in terms of time derivatives of disturbance torque signal T_d (Eq. 4). Parameter set $\mathbf{p}$ is determined using the general error function minimization method described in Chapter 2.2.2. From the basic variation relation (Eq. 5) error function derivative $\frac{\partial J}{\partial \mathbf{p}}$ reads as:

$$\frac{\partial J}{\partial \mathbf{p}} = \begin{Bmatrix} \dfrac{\partial J}{\partial K_P} \\ \dfrac{\partial J}{\partial K_I} \\ \dfrac{\partial J}{\partial K_D} \end{Bmatrix} = \begin{Bmatrix} \int_0^T \left((\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))u(t)\right)dt \\ \int_0^T \left((\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))\int_0^t u(\tau)d\tau\right)dt \\ \int_0^T \left((\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t))\dot{u}(t)\right)dt \end{Bmatrix}.$$

Partial derivatives $\frac{\partial\psi}{\partial u}, \frac{\partial\psi}{\partial\dot{u}}, \frac{\partial\psi}{\partial\ddot{u}}$ of under-integral of error function J calculated from Eq. (4) read as:

$$\frac{\partial\psi}{\partial u} = K_I;$$
$$\frac{\partial\psi}{\partial\dot{u}} = -(K_P - mgh);$$
$$\frac{\partial\psi}{\partial\ddot{u}} = K_D.$$

The time laws of conjugate variables time derivatives $\lambda(t), \dot{\mu}(t), \ddot{\eta}(t)$ are obtained from the Eqs. (6) and (7):

$$\begin{cases} \ddot{\lambda}m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \eta\ddot{\tilde{k}} = K_I; \\ \ddot{\mu}m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\dot{\tilde{k}} = -(K_P - mgh); \\ \ddot{\eta}m - \dot{\eta}\tilde{c} + \eta\tilde{k} = K_D. \end{cases} \quad (9)$$

They are solved together with initial conditions $\lambda_T = \dot{\lambda}_T = \dot{\mu}_T = \dot{\eta}_T = \mu_T = \eta_T$.

In order to minimize the error function J and solve Eq. **Klaida! Nerastas nuorodos šaltinis.** the *steepest descent* optimization method was used [12].

This procedure allows to identify models' disturbance torque T_d and parameters set **p** at the same time using only one COP signal, which was recorded during physical experiment.

3. Implementation

Body mass and height of COM are data of typical average adult male subject (taken form [4]): the mass moment of inertia I = 76 kgm^2, mass m = 60 kg and distance of COM from the ankle joint h = 0.87 m. The COP signal of the subject who has mass and height of the typical adult male was used recorded during physical experiments.

During those experiments there were recorded five COP signals by using sample rate of 100 Hz. The duration of the experiments was 60 s.

The CIP model parameters identification algorithms were implemented in Matlab7. Disturbance torque T_d signals used in parameters identification procedures were recalculated from from CIP model using Eq. 8 (Fig. 2).

The initial model parameters set **p** was chosen from Peterka [4] and recalculated in SI units. The initial set **p** is called "Center normal":

$$K_P = 1117.27 \text{ N m rad}^{-1}$$
$$K_I = 14.32 \text{ N m rad}^{-1}\text{s}^{-1}$$
$$K_D = 257.83 \text{ N m rad}^{-1}\text{s}$$

This set **p** according to [4] is able to produce a physiological COP signals.

The COM signal u which is used for T_d reconstruction is calculated from the COP signal passed through 4th order low-pass Butterworth filter as proposed by Benda et al [13]. The time derivatives of u are calculated by using numerical differentiation formulas as:

$$\dot{u} = \frac{u(t+\Delta t) - u(t-\Delta t)}{2\Delta t};$$
$$\ddot{u} = \frac{u(t+\Delta t) - 2u(t) + u(t-\Delta t)}{\Delta t^2}.$$
(1)

4. Results

All numerical experiments employing error function J minimization identified CIP model's parameters set **p**. Resuls of these experiments are presented in left column of Table 1.

Table 1. Identified sets **p**.

p	Number of Experiment				
	1	2	3	4	5
K_P	1118.12	1115.39	1119.75	1118.46	1117.21
K_I	15.15	13.95	15.69	14.95	14.42
K_D	255.44	253.44	257.73	258.23	257.10

Comparing obtained results with a "Center normal" set **p** presented in [4] it may be noticed that the identified sets are very close to the one provided by Peterka.

Statistical analysis of data provided in Table 1 revealed that all identified sets **p** may be treated as describing the same particular person.

5. Conclusions

The controlled inverted pendulum human posture model with single DOF was investigated. The error function minimization employing step-by-step procedure, when the gradient of the error function is used as the search direction was applied using error function based on disturbance signal.

Five numerical experiments were conducted in order to verify the ability of the method to identify the same set of CIP model parameters for a particular person.

The method demonstrated the ability to identify realistic CIP model parameters. All five identified sets may be treated as able to generate realistic COP signals.

All five identified sets **p** were close to the "Center normal" set of **p** from [4] and may be qualified as describing the same person regarding different disturbance torque signals present within the human body during physical experiments.

Further experiments must be conducted to ascertain that parameters set **p** and disturbance torque identification procedure gives different sets of **p** for the different persons.

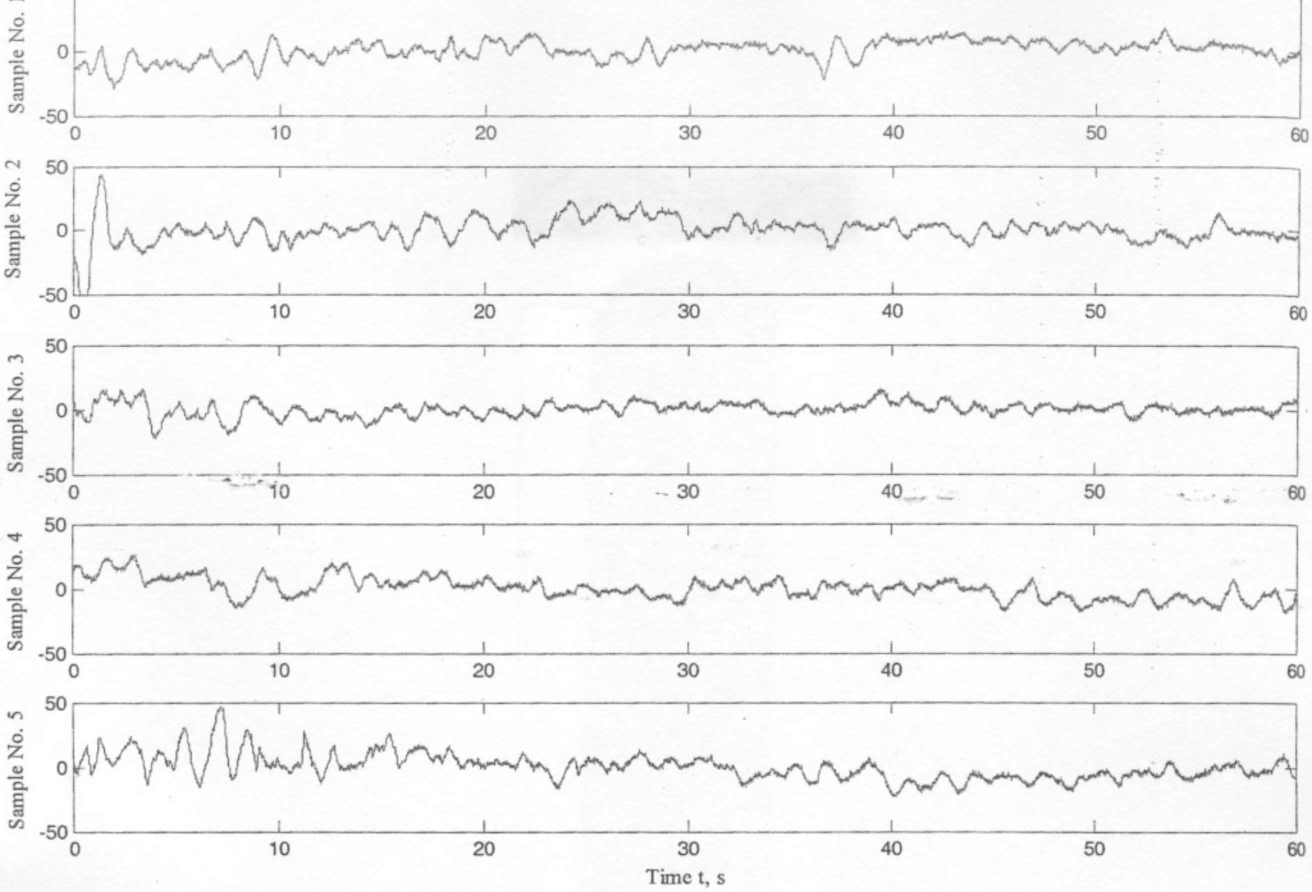

Fig. 2. Different samples of disturbance signal T_d, Nm

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